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BIBLIOGRAPHICAL NOTE ON THE USE OF THE WORD MASS IN CURRENT TEXTBOOKS.¹

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CONTENTS.

I. Introduction	1
II. Preliminary remarks on measurable quantities in general	2
III. Current definitions of the mass of a body:	
1. Mass as measured by force \div acceleration (inertia)	3
2. Mass as measured by a beam balance (standard weight)	4
3. Mass as measured by mutual acceleration (interaction)	6
4. Mass as measured by the number of identical corpuscles	7
IV. Incomplete "definitions" of mass:	
5. "Quantity of matter"	7
6. Circular definitions	9
7. Definition lacking	9
8. Mass as body, or lump of matter	9
V. Remarks on the names of the units of mass	9
VI. Conclusions:	
Propositions (A)–(G)	10
Proposition (H)	12
VII. List of books examined	12

I. INTRODUCTION.

The history of dynamics from before the time of Newton to the present day has been a history of continually increasing precision in the use of terms. Such words as power, energy, force, which only a few generations ago were hopelessly confused and ambiguous, are now well defined and accurately used by all careful writers.

Not quite all the ambiguities have been removed, however. In particular, the term "mass" is one which is still defined in a variety of ways—with much resulting confusion.

¹ Presented to the American Mathematical Society at its Cleveland meeting, September 5, 1917.

With a view to ascertaining exactly what meaning present-day students are being taught to attach to the word "mass," I have looked up the definition of the term in a large number of current books on mechanics and physics, and I venture to present here the result of this survey, in the hope that it may be not without interest to teachers and other mature students. Let me state most emphatically, however, that this bibliographical report is not intended for beginners, or for class-room use. *The chief end of a course in dynamics is not a definition of "mass," but the development of power in the solution of fundamental dynamical problems;* if too much time is spent on controversial matters, too little energy will be available for the mastery of essential principles.

II. PRELIMINARY REMARKS ON MEASURABLE QUANTITIES IN GENERAL.

Since we are not here concerned with metaphysical speculations as to the ultimate nature of matter, but only with a certain measurable property of matter which may be used in the equations of mechanics, it is important to recall briefly under what circumstances any given phenomenon can be treated as a measurable quantity. The whole progress of modern science has been a constant effort to reduce merely qualitative data to a basis of accurate quantitative measurement, and no concept (like the "food value" of a bag of flour, or the "amount of matter" in a lump of metal) can be regarded as having any scientific standing until some method of reducing that concept to measurement has been agreed upon.

A set of elements, a, b, c, \dots , is said to be measurable, in terms of a unit element, u , of the set, when the following conditions A, B, C , are satisfied:

(A) Given any two elements a, b , it must be possible to decide whether $a = b$, $a < b$, or $a > b$.

This condition implies the existence, among the elements, of a rule of comparison, $<$, which is supposed to obey all the ordinary laws of serial order.

(B) It must be possible to form multiples and sub-multiples of each element, or at least of the unit element, to any agreed-upon extent.

This condition implies the existence, among the elements, of a rule of combination, $+$, which is supposed to obey all the ordinary laws of addition in algebra.

(C) Given any element, a , it must be possible to find at least one positive integer, n , such that $(1/n)u \leq a$; and for every such value of n (within some prescribed range) it must be possible to find two consecutive positive integers, m and $m + 1$, such that

$$m \frac{u}{n} \leq a < (m + 1) \frac{u}{n}.$$

This condition implies that the rule of addition, $+$, and the rule of comparison, $<$, obey the law that if $x < y$ then $a + x < a + y$.

The wider the range of integers over which these conditions hold, the greater the accuracy attainable in the measurement.¹

¹ We may note, in passing, that many sets of entities satisfy condition A but not condition B . For example, in considering sensations of color, we say: this color is "less red" than that; but

Now it is obvious that *before the idea of "mass" can be used in the equations of dynamics, the "masses" of bodies must be defined as measurable quantities*; and all the different definitions of the word mass are in reality different definitions of the way in which the measurement of mass is supposed to be effected.

III. CURRENT DEFINITIONS OF MASS.

We now take up, in order, the four principal types of definitions of mass contained in the current textbooks. The books referred to are listed at the end of the article.

1. *Mass as Measured by Force per Acceleration (Inertia).*

In the opinion of J. Clerk Maxwell (p. 40), "the only definition of equal masses which can be admitted in dynamics" is the following: "Any two bodies are of equal mass if equal forces applied to these bodies produce, in equal times, equal changes in velocity"; and this definition is in fact the one adopted by the largest number of authors. Thus:

Lamb (p. 20): "It is observed . . . that the same force applied in succession to different bodies produces in general different degrees of acceleration. This is described as due to differences in the inertia or mass of the respective bodies. Two bodies which acquire equal velocities in equal times, when acted on by the same force, are regarded as dynamically equivalent, and their masses are said to be equal."

Poincaré (p. 13, tr.): "The quotient of force divided by acceleration is what we mean by the mass of a body."

E. H. Hall (p. 142): "Equal masses are, by definition, quantities of matter which, whatever their inequalities in other respects, are alike in this, that they require equal forces to give them equal velocities in equal times."

Similarly in Routh (p. 23), Poynting & Thomson (p. 4), Rankine (p. 482), Loney (p. 5), Cox (p. 118), Clifford (p. 59), Macgregor (p. 191), Garnett (p. 22), Williamson & Tarleton (p. 32), H. Adams (p. 10), Encyclopædia Britannica, Art. Mechanics (p. 974), New International Encyclopædia, Art. Matter (p. 1033), Lanza (p. 11), Trowbridge (p. 61), DuBois (p. 3), Wright (p. 46), Carhart (p. 42), Kimball (p. 18), Duff (p. 28), Slocum (p. 70), Ferry (p. 77), Mann & Twiss (p. 36), Thwing (p. 12), Trautwine (p. 336), Hayward (p. 194), Chwolson (p. 72), Deschanel (p. 44), Delauney (p. 119), Jamin (p. 62), Massau (p. 111), Ringelmann (p. 99), Violle (p. 98), La Grande Encyclopédie, Art. Masse, sub-heading Mécanique (p. 371), Autenrieth (p. 194), Hamel (p. 46), Streintz (p. 100), Voigt (p. 39), Voss (p. 49), Wüllner (p. 56), "Hütte" (p. 148), Weber & Wellstein (p. 93).

we hardly say: this color is exactly "twice as red" as that. Such entities may be said to be comparable, but not additive. On the other hand, many sets of entities satisfy *B* but not *A*. For example, two points in *n*-dimensional space have a readily definable "sum," but we do not speak of either of such points as being "less than" the other. Such entities may be said to be additive, but not comparable. No entities can be said to be strictly measurable unless they are both comparable and additive.

Also in the following books, if, as is clearly intended, we understand by "inertia" the ratio of force over acceleration: Tait (p. 28), Gray (p. 109), R. W. Stewart (p. 93), Morley (p. 27), Martin (p. 4), Slate (p. 224), Fuller & Johnston (p. 5), Culler (pp. 4, 20), Ames (1897, p. 31), Crew & Jones (p. 25), Carhart & Chute (p. 6), Duhem (p. 105).

This definition of mass as force \div acceleration is obviously unintelligible without some previous definition of the equality of two forces, and this is expressly pointed out by a number of writers. Thus:

Loney (p. 5): "It is here assumed that it is possible to create forces of equal intensity on different occasions, *e. g.*, that the force necessary to keep a given spiral spring stretched through the same distance is always the same when other conditions are unaltered."

R. W. Stewart (p. 92): "Equal forces may be defined as forces which extend the same spiral spring to the same extent."

Similarly, Voss (p. 51), Cox (p. 118), Chwolson (p. 70), Larmor (p. 278), and others.

Or, if preferred, equal forces may be defined as forces which give to the same lump of metal equal accelerations. In any case, it must be remembered that forces can be specified and compared without involving in any way the idea of the mass of a body.¹

(1a) A variation of this definition (by force over acceleration) is adopted by some writers who replace the general expression F/a by W/g , where W is the special force due to gravity, and g the corresponding acceleration. (Older writers use G/g for W/g .)

Thus I. P. Church (p. 53): "Since [for any given body] the quotient G/g is invariable, it will be used as the measure of the mass M in the body."

C. M. Woodward (p. 176): "The ratio W/g is quite universally represented by one letter m , which stands numerically for the mass of the body."

Similarly, Winkelmann (p. 4), Merriman (p. 148), Moulan (p. 129), Kent (p. 488), Larousse (p. 1310), Wood (p. 16), Worthington (p. 9), Perry (Calc., p. 26, Mech., p. 40), and probably Weisbach (p. 158), Hancock (p. 4), and Hudson (p. 74).

(1b) Another variation, expressed in terms of kinetic energy, is found in J. J. Thomson (pp. 28-30), Larmor (p. 181), Sanford (p. 27), Cheston, Gibson & Timmerman (p. 94), and Young (p. 108). Compare Holman, p. 61.

2. *Mass as Measured by a Beam Balance (Standard Weight).*

In spite of Maxwell's opinion that force divided by acceleration is the only admissible definition of mass, many writers in both pure and applied mechanics prefer a second definition, which we shall call the beam balance definition. Thus:

Franklin & MacNutt (pp. 25, 14): "The mass of a body, as a quantity, is

¹ Daniell's contention (p. 20) that "Force can never be measured until we know . . . the mass acted upon and the acceleration actually imparted to it" would appear to be refuted by the whole history of the science of statics.

defined by the operation of weighing by a balance. . . . The only proper definition of a quantity is the definition which corresponds to the fundamental method which is actually used in measuring that quantity. . . . It is all very well to *talk* about defining the mass of a body in accordance with the above utterly impracticable method of measuring its mass [by force and acceleration]; but sensible men always *define* things in physics in the way they *do* them."

Ganot (p. 15): "Two bodies are said to have equal masses when, if placed in a perfect balance in *vacuo*, they counterpoise each other."

Pender (p. 930): "Two bodies are said to have equal masses if, when they are suspended simultaneously in a vacuum, one from each end of an equal-armed balance, there is no tipping of the beam."

Capito (p. 170): "This is the only way in which the mass of a body can be found direct."

Similarly, in Love (p. 68), Maurer (p. 142), Stewart & Gee (p. 63), Daniell (p. 12), Nichols & Franklin (p. 8), Gage (p. 2), Henderson & Woodhull (p. 39), Hoadley (p. 15), Müller-Pouillet (p. 94).

Also in the following books, where "weighing" is clearly to be understood as "weighing on a beam balance": Thomson & Tait (p. 220), Jeans (p. 29), Lorentz (p. 118), Planck (p. 2), Minchin (p. 2), Newcomb (p. 554), Wentworth & Hill (p. 7), Dolbear (p. 19), C. F. Adams (p. 10), Everett (p. 16), Ziwet & Field (p. 120), Hastings & Beach (p. 5); and probably Miller & Lilly (p. 2), Black & Davis (pp. 154-155), and Föppl (p. 30).

The essential equivalence between the force \div acceleration definition and the beam balance definition is maintained by many writers. Thus

Kennelly (p. 8): "Mass [of a body] is estimated either by its inertia or by its weight."

Rowland & Ames (p. 46): "It is possible to define two objects as having equal masses if, when set in motion by the same cause, they have identical motions. [This is the force \div acceleration definition.] . . . Or, two bodies might be defined as being of equal mass if they have the same weight. [This is the beam balance definition.] . . . There is no a priori reason why there should be any connection between these two definitions; but it is found by experiment . . . that two bodies which have the same inertia also have the same weight. Consequently, it is immaterial which of these two properties is taken as the basis of comparison. . . . In practice masses are compared and measured by means of a balance, which is an instrument to measure weight."

It should not be forgotten, however, that the equivalence between these two definitions is merely an assumption, and indeed one which seems likely to be called into question by modern researches into the behavior of bodies moving with very high velocities. For a further discussion of the contrast between these two definitions, see § VI, *F*, below.

3. *Mass as Measured by Mutual Acceleration (Interaction).*

The following definition was first proposed by Mach in 1868,¹ and is associated in England with the name of Karl Pearson.

Mach (p. 218): "Those bodies are bodies of equal mass which, mutually acting on each other, produce in each other equal and opposite accelerations."

Pearson (p. 329): "We conceive a standard corpuscle, Q ; . . . then

$$\text{Mass of } A = \frac{\text{Acceleration of } Q \text{ due to } A}{\text{Acceleration of } A \text{ due to } Q}.$$

We have here a perfectly clear and intelligible definition of the mass of A relative to Q . It is in this manner that mass is invariably determined scientifically."

Appell (p. 89): "The ratio of the masses of two points is, by definition, the inverse of the ratio of the accelerations which each of them determines in the other; the numerical value of one mass having been chosen arbitrarily, the values of all the others are determined. The word force does not enter into the principles of dynamics as here developed."

Hoskins (p. 227): "The masses of two particles are in the inverse ratio of the accelerations which they give each other. This is a definition of mass."

Similarly, in Goodwill (p. 48), Encyclopædia Britannica, Art. Motion (p. 907), Barton (p. 196), Roberts (p. 72), Crew (Mech., p. 66, Physics, pp. 57-59), Ames (1904, p. 60), Lecornu (p. 207), Andrade (p. 54).

Also, as an alternative definition, in Auerbach (p. 36), and others.

The most obvious case of interaction between two bodies is the case of impact or collision, and hence most of the writers who use the mutual acceleration definition regard a "*collision balance*," that is, an apparatus for comparing velocities before and after collision, as the fundamental instrument for measuring masses. See, for example, Goodwill, pp. 50-53.

As to the relation between this definition and the preceding definitions: Love (p. 168) makes a distinction between the "mass-ratio" of two bodies, as determined by the mutual acceleration definition, and the "ratio of the masses" of the bodies, as determined by the beam balance definition, and asserts the equality of the two. Pearson (pp. 333-337) shows how, on certain assumptions, the mutual acceleration definition may be regarded as including both the beam balance definition and the force-per-acceleration definition as special cases. (Compare Love, pp. 168, 340.) On the other hand, if we accept the principle of action and reaction, the collision balance definition is obviously a special case of the force-per-acceleration definition, since the bodies are acted upon, during the brief interval of impact, by equal forces.

(3a) The purely analytic definitions of mass given by Kirchhoff (p. 23), Boltzmann (p. 21), Timerding (p. 287), and Webster (p. 23) may also be classed under this heading (of mutual acceleration).

¹ E. Mach, "Ueber die Definition der Masse," *Carl's Repertorium für Experimental-Physik*, Vol. 4, pp. 355-359 (1868).

These three types of definitions are the ones which the student is most likely to meet in his ordinary reading. (Compare The Century Dictionary, Art. Mass.)

4. *Mass as Measured by the Number of Identical Corpuscles in a Body.*

The following definition depends on a certain assumption in regard to the structure of matter.

Johnson's Cyclopædia, Art. Dynamics (p. 875): "If we suppose . . . that the ultimate particles or molecules of all substances are the same, and that we may designate by the term density the degree of proximity of the particles of any body to each other, then the number of particles in a given volume may be taken to denote the mass of the body."

Hertz (p. 46): "The number of material particles in any space, compared with the number of material particles in some chosen space at a fixed time, is called the mass contained in the first space."

La Grande Encyclopédie, Art. Masse, subheading Astronomie (p. 371, tr.): "One can then define the mass of a body as the number of identical particles (points matériels) of which it is composed."

Houston & Seal (p. 30): "The mass of a body is proportional to the number of its molecules."

This definition of mass, though seldom explicitly stated, is probably the idea that most authors have in mind when they speak vaguely of mass as "quantity of matter."

Granting the basic assumption, the definition is logically quite defensible. Unfortunately, however, there is not a shred of evidence in support of this assumption, and the student who has been brought up on any such notion of the ultimate structure of matter finds it difficult to adjust himself to the modern theories, which make the structure of the atom as complex as that of the solar system. And of course, as a practical method of determining the mass number of any given body, the definition fails completely, and resort must be had, sooner or later, to one of the other methods of measuring mass. (Compare Barton, p. 195.)

IV. INCOMPLETE DEFINITIONS OF MASS.

All the definitions so far mentioned satisfy the fundamental condition that masses must be measurable quantities. But a number of widely used text-books still retain "definitions" of mass which are no definitions at all, because they fail to explain how mass is to be measured.

5. *"Quantity of Matter."*

Almost every book on the subject pays its respects, in some form or other, to the time-honored "definition" of mass as "quantity of matter."

In compiling the present report, whenever I found this "definition" supplemented by a statement of how the "quantity of matter" is to be measured, I

ignored the words "quantity of matter" and accepted this supplementary statement as the author's real definition of mass. But where the words "quantity of matter" stand alone (as in Ziwet, p. 129), they must be regarded as a totally inadequate definition of the mass of a body as a quantitative concept. The situation is well described by many authors. For example:

Encyclopædia Britannica, Art. Motion (p. 907): "The mass of a body is often loosely defined as the measure of the quantity of matter in it. This definition correctly indicates that the mass of any portion of matter is equal to the sum of the masses of its parts, . . . but gives no test for comparison of the masses of bodies of different substances."

Jamin (p. 63, tr.): "Some authors define mass as the quantity of matter which the body contains. . . . This idea is vague and illusory, for matter is not a thing which one can measure. . . . Those who wish to preserve this idea of mass are obliged, at the start, to define what they propose to mean by equal quantities of matter."

Chwolson (p. 73, tr.): "The primitive definition of the term mass as quantity of matter is not admissible, for . . . for heterogeneous materials, the notion of equal or unequal quantities of matter is entirely lacking."

Hoskins (p. 2): "The mass of a body is often briefly defined as its quantity of matter. These words, however, convey no definite idea of the meaning of mass as a factor in the determination of motion."

It cannot be emphasized too strongly that without a definite statement of the method of measurement, the phrase "quantity of matter" is empty and useless in dynamics. *Merely calling a thing a quantity does not make it a quantity.* It is just as absurd to speak of "quantity" of matter without defining the method of measurement, as it would be to speak of "quantity" of beauty, or "quantity" of temperature, without defining some method of estimating the values of these "quantities."

There are, of course, familiar cases in which we do have a direct intuitive perception of comparative "quantity." For example, any one can tell that one speed is many times faster than another speed, or that one bulk is many times larger than another bulk. Speed and bulk are perfectly definite intuitive magnitudes, regardless of the method of measurement which may be used to determine their numerical value. *But in the case of matter, it is precisely this intuitive perception of quantitative relations which is entirely lacking.*

Suppose, for example, that a load of coal and a load of wood come to the door, and let us ask the question, which has more "matter" in it? It will be found, on reflection, that no answer to this question is forthcoming, *except* by reference to one or other of the measurable properties described above. We can tell at once which load has more bulk; we can hazard a guess as to which has more weight, or as to which would require more force to set it in motion; we can even amuse ourselves by pretending to count the number of identical particles of which we may imagine the two loads to consist; but when we try to think of any direct comparison between the mere "matter" in one load with the mere

"matter" in the other load, without making even subconscious use of any one of these methods of measurement, the mind becomes absolutely blank. No such direct comparison is possible, even in theory (except, of course, in the often cited but quite trivial and irrelevant case of the comparison of bodies of the same homogeneous material).

Fortunately for the student, the number of textbook writers who still persist in defining mass as *merely* "quantity of matter" is rapidly decreasing (since the publication of Pearson's *Grammar of Science*), and is already negligibly small.

6. *Circular Definitions.*

The definition of mass in terms of density, followed by a definition of density in terms of mass, still appears in Tait and Steele (p. 42), Bowser (p. 6), Andrews & Howland (p. 11). (Compare Thomson & Tait, p. 220.) The obvious circularity of this definition has been pointed out by many critics. Similarly, Dadourian (p. 103) defines mass in terms of "kinetic reaction," and on the same page defines "kinetic reaction" in terms of mass.

7. *Definition Lacking.*

A number of books in which one would expect to find a definition of mass use the term freely without formal definition of any kind. For example, Jamieson (p. 2), Millikan (p. 13), Smith & Longley (p. 90), Hedrick & Kellogg (p. 1), Watson (p. 68), Silberstein (p. 50).

(7a) On the other hand, some very successful writers prefer to develop the whole theory without using the *word* mass at all; so Cotterill & Slade (*loc. cit.*), Merriman (p. 148), Black & Davis (p. 154), and especially Sir George Greenhill (*loc. cit.*).

8. *Mass as Lump of Matter.*

One further use of the word mass may be mentioned, although it hardly pretends to be a definition of the mass of a body in any quantitative sense. When one speaks of "the impact of two masses," or when one solves a problem about "two masses suspended by a cord over a pulley," one obviously means by a *mass* simply a *body*, or *lump of matter*. This is a very common and convenient usage which ought not to be likely to create any misunderstanding.

This completes the record of all the important uses of the word mass in the current textbooks.

It remains to add a brief report on the various names that are given to the units of mass.

V. REMARKS ON THE NAMES OF THE UNITS OF MASS.

(1) If the mass of a body is defined by the force-per-acceleration definition, the name of the unit of mass is naturally derived from the names of the units of force and acceleration, just as the name of the unit of acceleration is itself

derived from the names of the units of length and time. Thus we speak of a mass of 1 lb. per ft.-per-sec.², or 1 kg. per cm.-per-sec.².

Worthington (p. 9) abbreviates 1 lb. per ft.-per-sec.² into 1 "slug," while Maurer (p. 143) calls this unit a "gee-pound."

On the other hand, some writers prefer to leave the unit unnamed altogether.

Church (p. 53): "No name will be given to the unit of mass, it being always understood that the fraction G/g will be put for M before any numerical substitution is made."

Similarly, Perry (Calc., p. 26), Sanford (p. 28).

(2) If the beam balance definition or the mutual acceleration definition is adopted, the name of the unit of mass is simply the name of the lump of metal which is used as the standard; as the standard pound body, the standard kilogram body, etc.

It must be remembered, however, that the words pound, kilogram, etc., are used also (quite properly) to denote units of force. Hence, in books in which forces and masses appear in the same equations, it is necessary to distinguish between the *pound force* and the *pound mass*, and between the *kilogram force* and the *kilogram mass*.

This distinction is emphasized by many writers, for example, Kennelly (p. 10). The attempt sometimes made, however, to symbolize this distinction by writing "lb." for one of the units and "pd." for the other seems foredoomed to failure, since no one can remember which is which. For example, Worthington (p. 9) uses "pd." for force and "lb." for mass, while Dadourian (p. 109) uses "pd." for mass and "lb." for force. Similarly, the two new notations recently proposed by Hudson (p. 74)—"pounds (abs.)" for mass as measured by weight, and "pounds (grav.)" for mass as measured by weight divided by g —would seem to serve only to increase the existing confusion.¹

In books in which forces are compared only with forces, and masses only with masses—so that no equation contains both forces and masses together—the necessity for this distinction disappears. This is the real secret of the simplicity, as regards units, of the method advocated by the present writer (see § VI, *H*, below).

VI. CONCLUSIONS.

In conclusion, I venture to express my own personal preference, in regard to the use of the word mass, by offering to defend the following propositions:

(4) It is impossible to assume, at the beginning of a course in mechanics,

¹ As an illustration of the confusion which arises when units of force and mass are introduced into the same equation, it may be amusing as well as instructive to reproduce the following statements from two books which have had a wider circulation perhaps than any others in their respective fields of theoretical and applied mechanics.

Routh (*Dynamics of a Particle*, p. 25): "The equation $W = mg$ shows that the weight of a unit mass is g ." (!) (One had always supposed that W was a force, and g an acceleration!)

Weisbach (*Mechanics of Engineering*, p. 159): "Hence the mass of a body whose weight is 20 pounds is 0.62 pounds; and inversely the weight of a mass of 20 pounds is 644 pounds." (!)

Quotations almost as distressing as these might be made from many more modern textbooks.

that a student has any intuitive idea of the meaning of the expression "the mass of a body." Before this term is introduced, its meaning should be defined (§ 7). The use of the word "mass" as a synonym for "lump of matter" is often convenient, and need not be discouraged (§ 8); but this gives, of course, no quantitative idea of the "mass of the body."

(B) The definition of "the mass of a body" as the number of identical corpuscles in it (§ 4), and the pseudo-definition of "mass" as "quantity of matter" (§ 5), are unsatisfactory, and should be abandoned.

(C) The definition of "mass" by mutual acceleration, at least in its general form (§ 3), is too abstract for beginners.¹

There remain to be considered, therefore, only the inertia definition (§ 1) and the beam balance definition (§ 2).

(D) The idea of the *inertia of a body*, as measured by force divided by acceleration, is important, and the Newtonian hypothesis that the inertia of a body is constant (that is, independent of the body's velocity) should be thoroughly understood. Any teacher who wishes to introduce the term "mass of a body" to mean the inertia of the body has excellent authority for doing so (§ 1).

(E) The idea of the *standard weight of a body*, as measured by a beam balance, is also important, and any teacher who wishes to introduce the term "mass of a body" to mean the standard weight of the body has excellent authority for doing so (§ 2).

(F) *It is by no means a matter of indifference, however, which of these two definitions of "mass" is adopted*, since many passages in standard literature presuppose one of these definitions to the absolute exclusion of the other.

As a first example, consider the following quotation from Poincaré (p. 13, tr.): "In the new mechanics, the mass of a body increases enormously with the velocity, and becomes infinite when the velocity approaches the velocity of light." Here the word "mass" clearly must mean inertia; the passage would become quite unintelligible if "mass" were interpreted as standard weight.—Secondly, consider the following proposition, cited by many authors as a fundamental law of nature: "Force = mass \times acceleration." Here the word "mass," whatever else it may mean, certainly does *not* mean inertia (that is, force \div acceleration), since, if it did, the proposition in question would be not a law of nature, but merely a trivial algebraic transformation of the very definition of inertia. To give the proposition any significance as a law of nature, the word mass must be interpreted as something other than inertia, for example, as standard weight.—Thirdly, consider the contrast between the two ways of naming the units. If the inertia definition is adopted, the natural units of mass are the lb. per ft.-per-sec.², the kg. per cm.-per-sec.², etc. (where "lb." and "kg." are to be understood as units of force); while if the beam balance definition is adopted, the natural units of mass are the pound-mass, the kilogram-mass, etc. (which are the names

¹ The collision balance, which is a very instructive apparatus, is best interpreted as an instrument for comparing the inertias of two bodies, the equality of the forces acting on the two bodies during the impact being insured by the principle of action and reaction (§ 3).

of certain lumps of metal); moreover, the latter units differ from the former by the numerical factors 32.1740 and 980.665 respectively.

These three illustrations may suffice to show that the choice between the two definitions of mass is a serious matter, which affects the whole development of the course.

(G) *In view of these considerations, it would appear desirable, at least at the beginning of the course, to employ only the separate, well-established terms "inertia of a body" and "standard weight of a body" to denote these two closely related but still quite distinct conceptions. Later in the course, the student should be told, as an important matter of general information, that the word "mass" is used by some writers to denote inertia and by other writers to denote standard weight. There is at any rate nothing to be lost by following this plan; and there is certainly much to be gained in the way of clear thinking.*

All these recommendations, it will be noticed, leave entirely open the question whether $F/a = F'/a'$ or $F = ma$ is the better form of the fundamental equation of mechanics, and I hope that a general acceptance of the truth of the propositions (A)–(G) may serve to clear the ground for a more satisfactory discussion of that question.

(H) To avoid possible misunderstanding, I should like to add that while I have constantly advocated the use of the equation $F/a = F'/a'$ as the fundamental equation of mechanics, I have no objection whatever to the *use of the letter m as an abbreviation for F/a , or W/g* . It is obvious, however, that the letter m , when so used, denotes an inertia, that is, a force-per-acceleration, and not a number derived from a beam balance. Hence, when numerical computation is involved, I like to replace such an m by the more explicit W/g before inserting numerical values, thus avoiding the possibility of confusion between the units of inertia and the units of standard weight. By following this plan, nothing is lost in the way of algebraic compactness, and much is gained in the way of sureness and comfort in numerical computation.¹—The alternative plan of regarding the " m " in $F = ma$ as a fundamental concept (usually vaguely defined as "quantity of matter") appears to me to offer no advantages, and to lead, in practice, to many disturbing and quite superfluous complications.

VII. LIST OF BOOKS EXAMINED.

This list includes merely those books on mechanics or physics which happened to come readily to hand, and is not intended to be in any sense exhaustive. The numbers in [] refer to sections of the present paper.

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J. S. AMES, 1897, *Theory of Physics*, Harpers, 1897 [1].

¹ For a systematic presentation of the working principles of mechanics, as developed from the fundamental equation $F/a = F'/a'$, see E. V. Huntington, *The Logical Skeleton of Elementary Dynamics*, AMERICAN MATHEMATICAL MONTHLY, Vol. 24 (1917), pp. 1–16. Reprints of this article, which a number of teachers have found useful in the class room, may be obtained from the Secretary, W. D. Cairns, 27 King Street, Oberlin, O., at ten cents a copy.

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- A. E. DOLBEAR, *First Principles of Natural Philosophy*, Ginn, 1897 [2].
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- A. W. DUFF, *Textbook of Physics*, Blakiston, 1908 [1].
- P. DUHEM, *Traité d'Énergétique*, I, Paris, 1911 [1].
- ENCYCLOPÆDIA BRITANNICA, 11th ed., vol. 17, Art. Mechanics, by W. J. M. RANKINE [1].
- ENCYCLOPÆDIA BRITANNICA, 11th ed., vol. 17, Art. Motion, Laws of, by W. H. MACAULAY [3, 5].
- LA GRANDE ENCYCLOPÉDIE, vol. 23, Art. Masse, subheading Mécanique [1].
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 A. E. KENNELLY, in F. F. FOWLE'S *Standard Handbook for Electrical Engineers*, 4th ed., McGraw, Hill, 1915 [2].
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Information in regard to errors of classification or omission will be gratefully received.

BOOK REVIEW.

SEND ALL COMMUNICATIONS TO W. H. BUSSEY, University of Minnesota.

Projective Geometry. By L. WAYLAND DOWLING. McGraw-Hill Book Company, New York, 1917. 215 pages. \$2.00.

If long and expectant waiting for the guest's arrival insures a hearty welcome, this little book first sees the light under most auspicious circumstances. For many years those who have had the good fortune to teach projective geometry have been wishing for a text in English that should lay sufficient emphasis on its non-metrical character and at the same time should be adapted to the powers of the average college junior or even the exceptional younger student. Cremona, excellent book though it is, does not satisfy the first condition; Holgate's translation of Reye has long been out of print; and Veblen and Young's masterful treatise has seemed to many too heavy for the purpose. Here we have a book, small and compact, of pleasing external appearance, well printed in good type, with clear and attractive page broken by figures of reasonable size. Its point of view is non-metrical and yet it does not neglect the metrical applications. It merits careful consideration by all who are interested in this beautiful field of mathematical thought and sympathetic trial in many of the institutions where the subject is taught.